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A simple-to-integrate formula of stress as a function of strain in concrete and its description procedure

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1. Introduction

When developing and researching structures it is often necessary to calculate the stress strain state of reinforced concrete members at sections perpendicular to the axis (at normal sections). The calculations have to be performed at different stages of loading:

- before cracking, (calculation of the stress of prestressed concrete, calculation of the losses of prestressing of reinforced concrete members);
- at cracking stage (defining the cracking moment);
- in concrete members with cracks at service stage (forecasting the width of cracks, the curvature and stiffness of the structures with cracks);
- at element failure stage (defining bearing strength of the element).

In the regulations [1], for the calculations of the parameters of these states simple but very conditional diagrams of the stresses are used:

- to calculate stresses before cracking triangular diagrams of elastic materials are taken;
- the cracking moments are calculated according to a very inaccurate pattern: in tensile zone – *rectangular* diagram, in compressed zone – *curvilinear* diagram that is not clearly defined;
- the empirical formulae for calculating crack width curvature and stiffness and the equations of the element strength were obtained having assumed a *rectangular* compressed concrete stresses diagram; the tensioned concrete above the crack tensioning is absolutely disregarded here.

In the missing of a uniform calculation method and when calculation patterns are so grossly distorted it is assumed:

- numerous correction coefficients need to be used;
- the empiric formulae of these correction coefficients are sometimes very bulky and cumbersome and devising of such formulae is costly;
- there is no possibility to calculate the height of the cracks and the height of tensile zones of the concrete above them;
- the height of compressive zone of the concrete is defined with a very low degree of accuracy;
- it is impossible to calculate stress strain state of reinforced members according to the measured parameters for the stages before cracking and before failure;
- it is impossible to define stress strain state in plastic hinges of the continuity beams, i.e. when the strength of the member decreases due to the fact that the stress of the reinforcement exceeds yield limit or/and the stresses of the compressive zone of concrete exceed the strength limit of concrete;
- there is no method of defining stress strain state of reinforced members according to the measured parameters of the cracks (the height and the width of the cracks and the distance between the cracks) in stages when their loads exceed the service loads.

In the absence of a general calculation method and when so grossly distorted calculation patterns are assumed, the calculation has a lot of shortcomings.

It is possible to solve many of these problems using more realistic curvilinear diagrams of interrelation between strain and stress of concrete. Such a curvilinear diagram is presented in the regulations [1, 2]. But its use is aggravated by the integral calculation problems.

Unfortunately, there exists no such universal but user-friendly method that would directly use nonlinear stress strain relation without replacing it, for instance, by broken lines comprised of segments of a straight line [3] or would take into account the possible deviations of the strains of the materials from the plane sections hypothesis [4].

In the calculation methods proposed by the author [5, 6] simple-to-integrate *curvillinear* diagrams of materials are used ant it is possible to take into account the deviations of the strains from the *plane sections hypothesis.*

The relevance of the use of curvillinear diagrams $\sigma-\varepsilon$ is proved by research papers of numerous authors [7–15].

The objective of the present paper is to describe the most general curvilinear diagram of interrelation between strain and stress of concrete presented in the the regulations [1, 2] Eq. (1) so that it would be easy to integrate and would be applicable to the concrete of strength classes C8/10-C90/105.

2. Object of the work

The present paper deals with the normal, heavy and fine-grained concrete of classes C8/10-C90/105.

The key designations are the same as in the regulations [1]: σ_c is normal stresses of compression of concrete; f_{cm} is average compression stress of concrete; f_{ck} is characteristic concrete crushing stress; E_{cm} is concrete secant modulus of elasticity; ε_c is strain of concrete; ε_{c1} is strain of concrete corresponding to f_{cm}; ε_{cu1} is limiting strain/deformation of concrete.

SI units used in the paper:

- of stress – MPa;
- of elastic modulus – GPa;
- of strain – %.

Fig. 1 shows curvilinear interrelation between strain and stress $\sigma-\varepsilon$ of concrete specified in the concrete and reinforced concrete design regulations [1] adopted for the eurocode area [2].

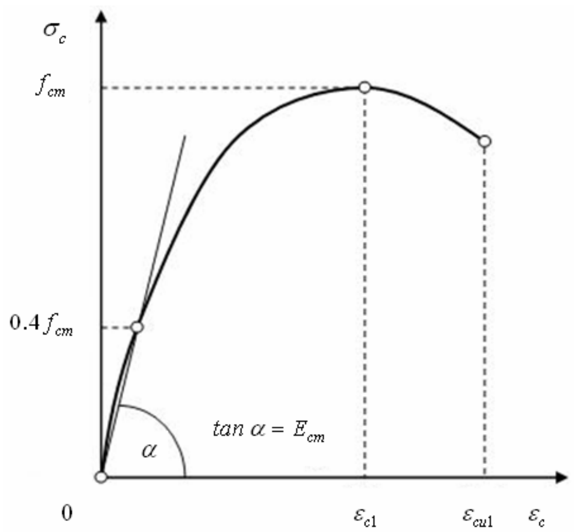

Fig. 1 Stress strain interrelation of concrete (Eq. 1)

This σ–ε interrelation is described by expressions (1-3)

$$\frac{\sigma_c}{f_{cm}} = \frac{k\eta - \eta^2}{1 + (k-2)\eta} \tag{1}$$

$$\eta = \varepsilon_c / \varepsilon_{c1} \tag{2}$$

$$k = \frac{1.1 E_{cm} \varepsilon_{c1}}{f_{cm}} \tag{3}$$

Here and hereafter the values of ε_{c1} and σ_c are considered to be positive. If $E_c = 1{,}1E_{cm}$ and $\nu_{c1} = f_{cm} / E_c \varepsilon_{c1}$, then $k = 1/\nu_{c1}$ and $\sigma_c = \dfrac{\eta/\nu_{c1} - \eta^2}{1 + (1/\nu_{c1} - 2)\eta} f_{cm}$.

Eq. (1) is simple, but is unhandy when we have to integrate curvilinear diagrams of stresses.

In the methodology proposed by us [5, 6] for the calculation of the parameters of stress of strain of reinforced concrete at normal sections a curvilinear $\sigma - \varepsilon$ diagram is used which is described by the following formula

$$\sigma_c = \nu_c E_c \varepsilon_c = \nu_c \sigma_{ce} \tag{4}$$

In Eqs. (4-9)

$$\nu_c = 1 + c_1\eta + c_2\eta^2 + c_3\eta^3 + c_4\eta^4 \tag{5}$$

$E_c = tan\,\beta;$ $E_r = tan(-\gamma)$; $e_r = E_r / E_c$; $\sigma_{ce} = E_c\varepsilon_c$; $\sigma_{ce1} = E_c\varepsilon_{c1}$; $\sigma_{c1} = f_{cm}$; $\nu_{c1} = \sigma_{c1}/\sigma_{ce1}$; $\nu_r = \sigma_{cr}/\sigma_{cre}$; $n_r = \nu_r/\nu_{c1} = \beta_r/\eta_r$; $\beta_r = \sigma_{cr}/\sigma_{c1}$; $\eta_r = \varepsilon_{cr}/\varepsilon_{c1}$. For values β, γ and for values of indexes see Fig. 2.

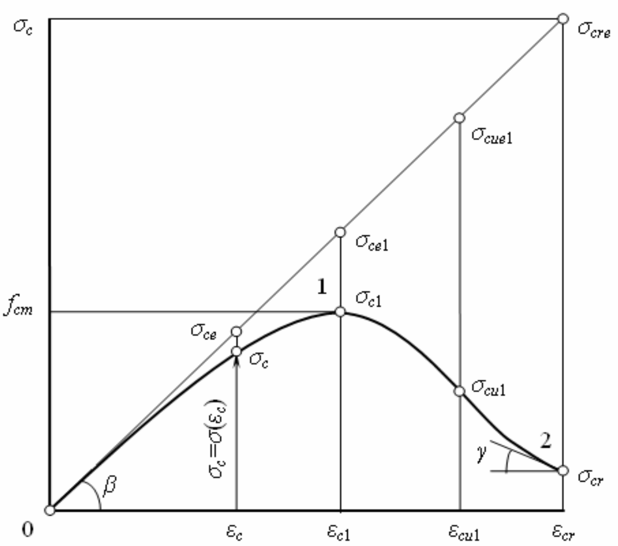

Fig. 2 Interrelation between strain and stress of concrete (Eq. 4)

Values of coefficients c_i

$$c_1 = \{2(\eta_r+1)(1-\eta_r)^3 - (\eta_r-1)e_r + [(5\eta_r-3)n_r + (3\eta_r-5)\eta_r^3]\nu_{c1}\} / [\eta_r(\eta_r-1)^3] \tag{6}$$

$$c_2 = \{(\eta_r^2+4\eta_r+1)(\eta_r-1)^3 + (2\eta_r^2-\eta_r-1)e_r + 2[(1+\eta_r-5\eta_r^2)n_r + (5-\eta_r-\eta_r^2)\eta_r^3]\nu_{c1}\} / [\eta_r^2(\eta_r-1)^3] \tag{7}$$

$$c_3 = \{2(\eta_r+1)(1-\eta_r)^3 - (\eta_r^2+\eta_r-2)e_r + [(5\eta_r^2+5\eta_r-4)n_r + (4\eta_r^2-5\eta_r-5)\eta_r^2]\nu_{c1}\} / [\eta_r^2(\eta_r-1)^3] \tag{8}$$

$$c_4 = \{(\eta_r-1)^3 + (\eta_r-1)e_r + 2[(1-2\eta_r)n_r + (2-\eta_r)\eta_r^2]\nu_{c1}\} / [\eta_r^2(\eta_r-1)^3] \tag{9}$$

3. Calculation procedure of stresses σ_c employing the proposed method

Assuming that $k_2 = k - 2$, $\eta = \varepsilon_c/\varepsilon_{c1} = \eta_{11}\varepsilon_c$, $\eta_{11} = 1/\varepsilon_{c1}$, $\beta_c = \sigma_c / f_{cm}$, from Eq. (1) we get

$$\sigma_c = \frac{f_{cm}(k-\eta)\eta}{1 + k_2\eta} \tag{10}$$

$$\eta = \frac{k - \beta_c k_2}{2} \pm \sqrt{\frac{(k-\beta_c k_2)^2}{4} - \beta_c} \tag{11}$$

$$E_t = \frac{d\sigma_c}{d\varepsilon_c} = \frac{f_{cm}\eta_{11}(k - 2\eta - k_2\eta^2)}{(1+k_2\eta)^2} \tag{12}$$

The values of E_t calculated using Eq. (12) in the interval $\varepsilon_c < \varepsilon_{c1}$ are positive, and in the interval $\varepsilon_c > \varepsilon_{c1}$ they are negative.

When $\beta_c = 0$, i.e. $\sigma_c = 0$, then $\eta = k$.

Using the formulae given in the regulations [1] we get the following values of the parameters

$$f_{cm} = f_{ck} + 8 \tag{13}$$

$$E_{cm} = 22(f_{cm}/10)^{0.3} \tag{14}$$

$$E_c = 1.1 E_{cm} \tag{15}$$

$$\varepsilon_{c1} = 0.7 f_{cm}^{0.31} \tag{16}$$

$$\begin{aligned} &\varepsilon_{cu1} = 3.5, \text{ if } f_{ck} \le 50 \text{ MPa} \\ &\varepsilon_{cu1} = 2.8 + 27[(98 - f_{cm})/100]^4, \text{ if } f_{ck} \ge 50 \text{ MPa} \end{aligned} \tag{17}$$

The next step is as follows:

– it is assumed that $\varepsilon_{cr} = \varepsilon_{cu1}$; then $\eta_r = \varepsilon_{cr}/\varepsilon_{c1} = \varepsilon_{cu1}/\varepsilon_{c1}$; when η_r < 1,1, then instead of the actual value of η_r we assume that η_r = 0,5;
– according to $\eta = \eta_r$ from Eq. (10) we get $\sigma_c = \sigma_{cr}$ and using Eq. (12) we get the value $E_t = E_r$ of the tangential modulus;
– we calculate $\beta = \beta_r = \sigma_{cr}/\sigma_{c1} = \sigma_{cr}/f_{cm}$, $n_r = \nu_r/\nu_{c1} = \beta_r/\eta_r$, $e_r = E_r/E_c$ and $\nu_{c1} = f_{cm}/E_c\varepsilon_{c1}$;
– using Eq. (4) we calculate the values of σ_c.

For the calculation of the values of stress σ_c using Eq. (1) and Eq. (4), the author of the present paper has developed respective programmes. When calculating σ_c the value of only one parameter f_{ck} needs to be entered in the programs.

The comparison of the results of stress calculation σ_c according to Eq. (1) of the regulations [1] and Eq. (4) proposed by the author of the present paper is illustrated in Fig. 3 and in the Table. The results of the calculation of strain ε_c in the interval from zero till ε_{cu1}, (in Fig. 3 the limit ε_{cu1} is marked by vertical lines in the graph, and by bold numbers in the Table) according to both methods are very close: the maximum difference is 1.5 %. The results are also close when $\varepsilon_c > \varepsilon_{cu1}$.

We have also analysed the values of σ_c for the concretes whose f_{ck} = (5-8) MPa and f_{ck} = (90-130) MPa, that have been similarly calculated using Eq. (1) and Eq. (4). Good results have been obtained.

4. Examples of the curvilinear diagram use

We illustrate the possibility of using the curvilinear diagram by an example of the calculation of the reinforcement and bearing strength of a reinforced concrete beam.

The cross-section to be calculated and the forces acting in it are shown in Fig. 4. The stresses of the concrete in tensile zone are disregarded. The stresses of the concrete in compressive zone are calculated from Eq. (4). The resultant force N_c of these stresses and its moment M_c in respect of the neutral axis $0-0$ is calculated from the Eqs. (18) and (19)

$$N_c = \int_{x_w}^{0} \sigma_c b(dx_c) = \omega\varepsilon_w E_c b x_w = \omega\varepsilon_w E_c b d \xi_w \tag{18}$$

$$M_c = \int_{x_w}^{0} \sigma_c b x_c (dx_c) = \varpi\varepsilon_w E_c b x_w^2 = \varpi\varepsilon_w E_c b d^2 \xi_w^2 \tag{19}$$

In Eqs. (18), (19) (see [5 or 6])

$$\omega = \frac{1}{2} + \frac{c_1}{3}\eta + \frac{c_2}{4}\eta^2 + \frac{c_3}{5}\eta^3 + \frac{c_4}{6}\eta^4 \tag{20}$$

$$\varpi = \frac{1}{3} + \frac{c_1}{4}\eta + \frac{c_2}{5}\eta^2 + \frac{c_3}{6}\eta^3 + \frac{c_4}{7}\eta^4 \tag{21}$$

$$\eta = \frac{\varepsilon_w}{\varepsilon_{c1}},\ c_i \text{ – from Eqs. (6)-(9)},\ \xi_w = \frac{x_w}{d}.$$

Eccentricity e_c of force N_c in respect of the neutral axis $0-0$ is as follows

$$e_c = \frac{M_c}{N_c} = \frac{\varpi}{\omega} x_w = \frac{\varpi}{\omega}\xi_w d \tag{22}$$

When the plane-sections hypothesis is assumed, then

$$\frac{\varepsilon_w}{\varepsilon_s} = \frac{x_w}{d - x_w} = \frac{\xi_w}{1 - \xi_w} \tag{23}$$

and the force of the reinforcement

$$\begin{aligned} N_s &= \sigma_s A_s = \varepsilon_s \nu_S E_s A_s = \nu_S E_s A_s \frac{d - x_w}{x_w}\varepsilon_w = \\ &= \nu_S E_s A_s \frac{1 - \xi_w}{\xi_w}\varepsilon_w \end{aligned} \tag{24}$$

In Eq. (24) E_s is the module of elasticity of reinforcement and $\nu_S = \sigma_s/\varepsilon_s E_s$ is the coefficient of elasticity of reinforcement.

Interval from N_c to N_s

$$\begin{aligned} z_c &= d - a_c = d - \left(1 - \frac{\varpi}{\omega}\right) d\xi_w = \\ &= \left[1 - \left(1 - \frac{\varpi}{\omega}\right)\xi_w\right] d \end{aligned} \tag{25}$$

where

$$a_c = x_w - e_c = d\xi_w - \frac{\varpi}{\omega} d\xi_w = \left(1 - \frac{\varpi}{\omega}\right) d\xi_w \tag{26}$$

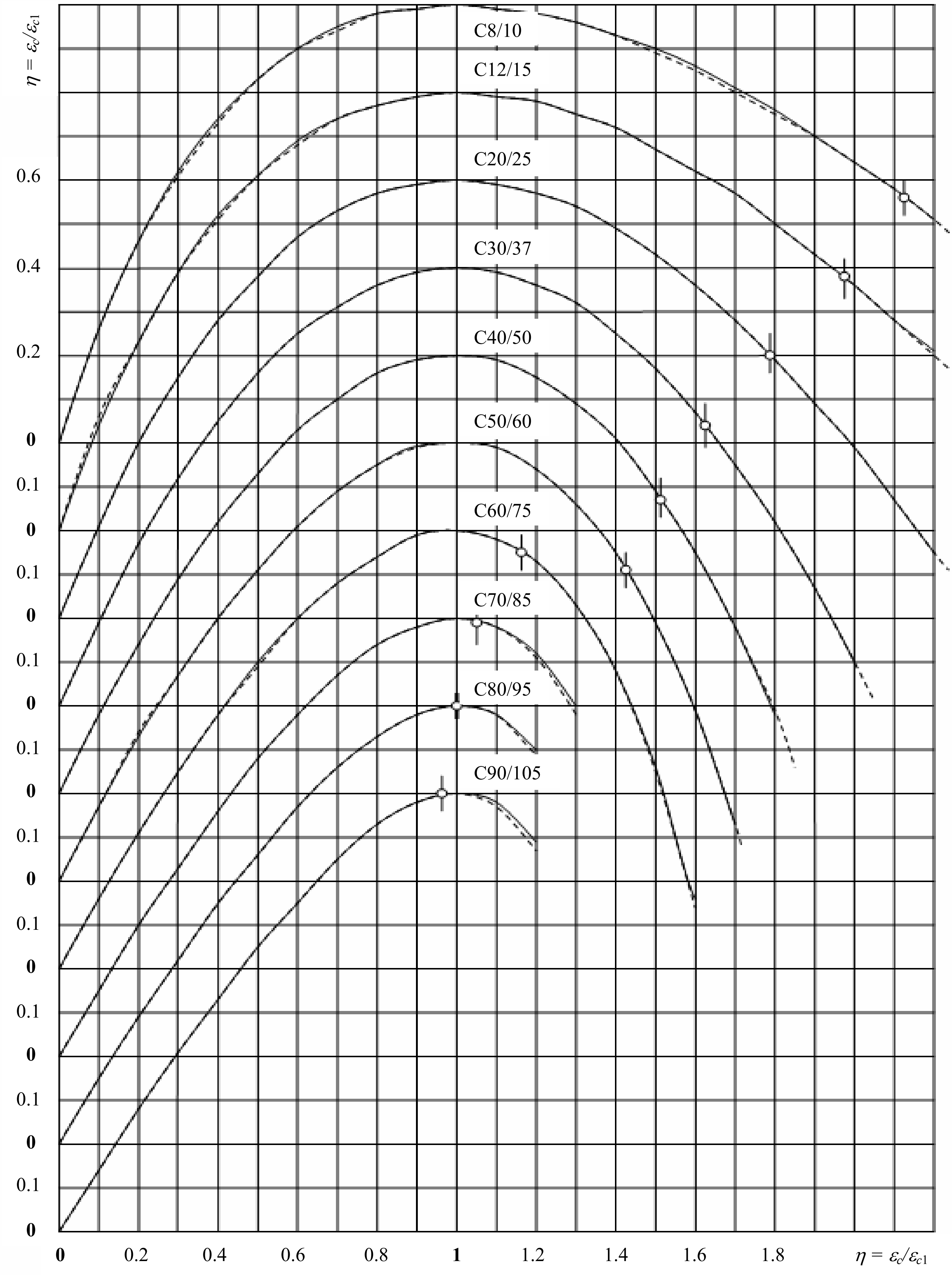

Fig. 3 Graphs of the relative values $\beta = \sigma_c / f_{cm}$ of stresses σ_c: ------ $\beta_{STR} = \sigma_c / f_{cm}$, when σ_c calculated using Eq. (1); —— $\beta_{ZI} = \sigma_c / f_{cm}$, when σ_c calculated using Eq. (4)

Table

Comparison of the results of the calculation of stresses σ_c received using the proposed method and the method given in the regulations

Concrete class	Calculation method	$\eta = \varepsilon_c / \varepsilon_{c1}$																						
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2
C8/10	$\beta_{ZI}*10^3$	0	258	462	621	743	835	902	948	978	995	1000	995	982	962	934	899	858	811	758	701	639	575	**511**
	$\beta_{STR}*10^3$	0	256	456	612	734	826	895	944	977	994	1000	995	981	958	928	892	850	804	752	697	638	575	**509**
	β_{ZI}/β_{STR}, |%|	0	0.8	1.3	**1.5**	1.2	1.1	0.8	0.4	0.1	0.1	0.0	0.0	0.1	0.4	0.6	0.8	0.9	0.9	0.8	0.6	0.1	0.0	**0.4**
C12/15	$\beta_{ZI}*10^3$	0	240	435	592	717	814	887	939	974	994	1000	994	978	952	917	875	824	767	703	635	**561**	485	406
	$\beta_{STR}*10^3$	0	239	433	589	713	811	884	938	974	994	1000	994	977	951	916	872	822	765	703	634	**561**	484	402
	β_{ZI}/β_{STR}, |%|	0	0.4	0.5	0.5	**0.6**	0.4	0.3	0.1	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.3	0.2	0.3	0.0	0.2	**0.0**	0.2	1.0
C16/20	$\beta_{ZI}*10^3$	0	226	415	570	697	798	875	932	971	993	1000	993	974	944	903	852	793	725	649	**567**	479	385	288
	$\beta_{STR}*10^3$	0	226	414	569	696	797	875	932	971	993	1000	993	974	944	903	852	792	725	649	**567**	478	384	283
	β_{ZI}/β_{STR}, |%|	0	0.0	**0.2**	**0.2**	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	**0.0**	0.2	0.2	1.8
C20/25	$\beta_{ZI}*10^3$	0	215	398	553	681	784	866	926	968	992	1000	993	971	936	889	830	761	681	**592**	493	387	273	152
	$\beta_{STR}*10^3$	0	215	398	552	680	784	865	926	968	992	1000	993	971	936	889	830	761	681	**592**	493	386	272	149
	β_{ZI}/β_{STR}, |%|	0	0.0	0.0	**0.2**	0.1	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	**0.0**	0.0	0.3	0.4	2.0
C30/37	$\beta_{ZI}*10^3$	0	194	367	518	648	757	845	913	962	991	1000	991	963	916	851	769	669	**552**	318	267	099		
	$\beta_{STR}*10^3$	0	194	367	518	648	757	845	913	962	991	1000	991	963	916	851	769	669	**552**	418	267	099		
	β_{ZI}/β_{STR}, |%|	**0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**		
C40/50	$\beta_{ZI}*10^3$	0	180	344	491	621	734	827	902	956	989	1000	988	953	893	806	**693**	551	379	176				
	$\beta_{STR}*10^3$	0	180	344	491	621	733	827	901	956	989	1000	988	953	893	806	**693**	551	379	176				
	β_{ZI}/β_{STR}, |%|	0	0.0	0.0	0.0	0.0	**0.1**	0.0	**0.1**	**0.1**	**0.1**	0.0	**0.1**	**0.1**	**0.1**	0.0	**0.0**	0.0	0.0	0.0				
C50/60	$\beta_{ZI}*10^3$	0	169	326	470	599	714	811	891	950	987	1000	986	942	865	750	**593**	388	131					
	$\beta_{STR}*10^3$	0	169	325	469	599	713	811	890	950	987	1000	986	942	865	750	**593**	388	129					
	β_{ZI}/β_{STR}, |%|	0	0.0	**0.3**	0.2	0.0	0.1	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	**0.0**	0.0	1.5					
C55/67	$\beta_{ZI}*10^3$	0	164	318	461	590	705	804	885	947	986	1000	985	936	**848**	716	531	285						
	$\beta_{STR}*10^3$	0	164	317	459	589	704	803	885	947	986	1000	985	936	**848**	715	530	280						
	β_{ZI}/β_{STR}, |%|	0	0.0	0.3	**0.4**	0.2	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	**0.0**	0.1	0.2	1.8						
C60/75	$\beta_{ZI}*10^3$	0	160	311	452	581	696	797	881	944	985	1000	983	**929**	830	677	460							
	$\beta_{STR}*10^3$	0	160	310	450	579	695	796	879	944	985	1000	983	**929**	830	675	452							
	β_{ZI}/β_{STR}, |%|	0	0.0	0.3	**0.4**	0.3	0.1	0.1	0.2	0.0	0.0	0.0	0.0	**0.0**	0.0	0.3	1.8							
C70/85	$\beta_{ZI}*10^3$	0	152	297	434	562	678	781	869	938	983	1000	**981**	918	800									
	$\beta_{STR}*10^3$	0	152	297	434	562	678	781	869	938	983	1000	**980**	914	785									
	β_{ZI}/β_{STR}, |%|	**0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	0.4	1.9									
C80/95	$\beta_{ZI}*10^3$	0	146	286	420	546	662	768	859	932	982	**1000**	978	904										
	$\beta_{STR}*10^3$	0	146	286	420	546	662	768	859	932	981	**1000**	977	894										
	β_{ZI}/β_{STR}, |%|	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	**0.1**	**0.0**	0.1	1.1										
C90/ /105	$\beta_{ZI}*10^3$	0	140	277	407	532	648	755	849	926	980	**1000**	975	889										
	$\beta_{STR}*10^3$	0	140	276	407	531	648	755	848	925	979	**1000**	972	869										
	β_{ZI}/β_{STR}, |%|	0	0.0	**0.4**	0.0	0.2	0.0	0.0	0.0	0.1	0.1	**0.0**	0.3	2.3										

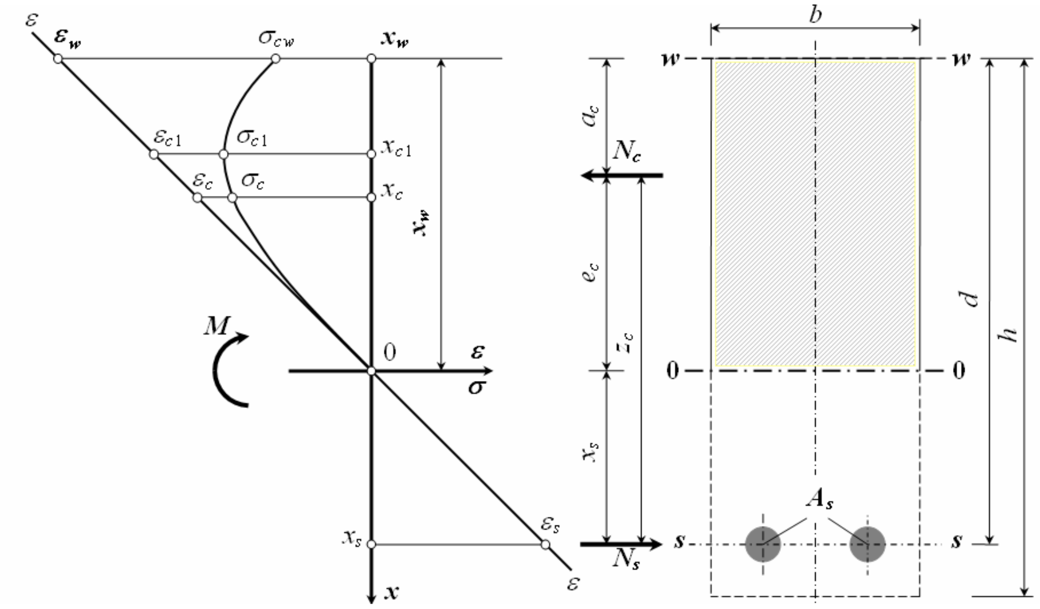

Fig. 4 The cross-section to be calculated and the acting forces

The moment of force N_c in respect of axis $s-s$

$$M_s = N_c z_s = \omega \varepsilon_w E_c b d \xi_w \left[1-\left(1-\frac{\varpi}{\omega}\right)\xi_w\right] d =$$

$$= [\omega\xi_w - (\omega-\varpi)\xi_w^2]\varepsilon_w E_c b d^2 \qquad (27)$$

The conditions of static equilibrium of axial forces and bending moments in respect of the axis $s-s$ of the centre (point) of gravity of the reinforcement

$$\omega\varepsilon_w E_c b d \xi_w - \nu_S E_s A_s \frac{1-\xi_w}{\xi_w}\varepsilon_w = 0 \qquad (28)$$

$$\omega\varepsilon_w E_c b d \xi_w^2 + \nu_S \varepsilon_w E_s A_s \xi_w - \nu_S \varepsilon_w E_s A_s = 0 \qquad (28a)$$

$$M - [\omega\xi_w - (\omega-\varpi)\xi_w^2]\varepsilon_w E_c b d^2 = 0 \qquad (29)$$

$$(\omega-\varpi)\varepsilon_w E_c b d^2 \xi_w^2 - \omega\varepsilon_w E_c b d^2 \xi_w + M = 0 \qquad (29a)$$

When the plane-sections hypothesis is disregarded and it is assumed that $\varepsilon_w = \varepsilon_{cu1}$ and $\sigma_s = f_s$ (here f_s is stress of reinforcement), simpler equations are received

$$\omega\varepsilon_w E_c b d \xi_{cu1} - f_s A_s = 0 \qquad (30)$$

$$M = M_s = N_c z_c = [\omega\xi_w - (\omega-\varpi)\xi_w^2]\varepsilon_{cu1} E_c b d^2 \qquad (31)$$

For example, dimensions of the rectangular cross-section shown in Fig. 4: $b = 0.20$ m; $h = 0.50$ m; $d = 0.46$ m; strength class of concrete C25/30 $\sigma_{c1} = f_{cm} = 33$ MPa; $E_{cm} = 31.476$ GPa; $\varepsilon_{c1} = 2.0694$ ‰; $\varepsilon_{cu1} = 3.5$ ‰; reinforcement $S400$: $f_s = f_{sk} = 400$ MPa; $E_s = 200$ GPa.

$$E_c = 1.1 E_{cm} = 1.1 \cdot 31.476 = 34.623 \text{ GPa},$$

$$\nu_{c1} = \frac{\sigma_{c1}}{E_c \varepsilon_{c1}} = \frac{33}{31.476 \cdot 2.0694} = 0.5066\,,$$

$$\alpha_{es} = E_s / E_c = 200/34.623 = 5.7765\,.$$

When $\varepsilon_w = \varepsilon_{cu1} = 3.5$ ‰, then

$\eta = \dfrac{\varepsilon_{cw}}{\varepsilon_{c1}} = \dfrac{3.5}{2.0694} = 1.6913$, and from Eqs. (6-9):

$c_1 = -0.6311$, $c_2 = 0.1059$, $c_3 = -0.01559$ and $c_4 = 0.001389$.

From Eqs. (20) and (21) $\omega = 0.20674$ and $\varpi = 0.11622$.

4.1. Calculation of the area A_s of the reinforcement, when the bending moment $M = 240$ kN·m

$$c1 = \varepsilon_w E_c b d = 3.5 \cdot 34.623 \cdot 0.20 \cdot 0{,}46 = 11.1486 \text{ MN};$$

$$c = \varepsilon_w E_c b d^2 = 3.5 \cdot 34.623 \cdot 0.20 \cdot 0.46^2 = 5.1284 \text{ MN·m};$$

$$\omega - \varpi = 0.20674 - 0.11622 = 0.09052\,.$$

From Eq. (29a) we calculate the value of ξ_w

$$(0.20674-0.11622)\cdot 3.5\cdot 34.623\cdot 0.20\cdot 0.46^2\cdot\xi_w^2 +$$

$$-0.20674\cdot 3.5\cdot 34.623\cdot 0.20\cdot 0.46^2\cdot\xi_w + 240\cdot 10^{-3} = 0;$$

$$0.09052\cdot 5.1284\cdot\xi_w^2 - 0.20674\cdot 5.1284\cdot\xi_w + 240\cdot 10^{-3} = 0;$$

$$0.46422\cdot\xi_w^2 - 1.06025\cdot\xi_w + 240\cdot 10^{-3} = 0\,;$$

$$\xi_w^2 - 2.28394\cdot\xi_w + 0.51700 = 0\,;$$

$$\xi_w = \frac{2.28393}{2} \pm \sqrt{\left(\frac{2.28393}{2}\right)^2 - 0.51700} =$$

$$= 1.14197 \pm 0.88718 = 0.25479$$

$$x_w = \xi_w d = 0.25479 \cdot 0.46 = 0.11720 \text{ m}$$

From Eq. (30) we calculate the area A_s of cross-section of the reinforcement

$$A_s = \omega\varepsilon_{cu1}E_c bd\xi_w / f_s =$$
$$= 0.20674 \cdot 3.5 \cdot 34.623 \cdot 0.20 \cdot 0.46 \cdot 0.25479 / 400 =$$
$$= 14.681 \cdot 10^{-4}\,\text{m}^2$$

The reinforcement rate

$\rho_l = 14.681 \cdot 100/(20 \cdot 46) = 1.596\,\%$.

4.2. Calculation of the bending moment M when the area of the reinforcement $A_s = 14.681 \cdot \text{cm}^2$.

From Eq. (30) we calculate ξ_w

$$\xi_w = \frac{f_s A_s}{\omega\varepsilon_{cu1}E_c bd} = \frac{400 \cdot 14.681 \cdot 10^{-4}}{0.20674 \cdot 11.1486} = 0.25478.$$

From Eq. (31) we calculate the bending moment:

$$M = [\omega\xi_w - (\omega - \varpi)\xi_w^2]\,\varepsilon_w E_c bd^2 =$$
$$= [0.20674 \cdot 0.25478 - 0.09052 \cdot 0.25478^2] \cdot 5.1284 =$$
$$= 0.2399952 \quad \text{MN}\cdot\text{m} \approx 240 \quad \text{kN}\cdot\text{m}.$$

5. Conclusions

For the calculation of stresses σ_c of heavy and fine-grained concrete of strength classes C8/10-C90/105 within the strain interval ε_c the interval from zero till ε_{cu1} instead of the equation adopted in the regulations [1] (in our paper it is marked by number (1)) Eq. (4) can be used. It is reasonable to do this when there is a need to perform integral calculation curvilinear diagram of stresses.

The results of calculation of stresses σ_c using Eqs. (1) and (4) are also close for concretes e f_{ck} = (5-8) MPa and f_{ck} = (90-130) MPa.

When the stress strain relation is described in an easy-to-integrate form such as Eq. (4), a lot of things may be calculated using the method proposed, for instance, in paper [6].

References

1. STR 2.05.05:2005. Design of Concrete and Reinforced Cnstructions.-Vilnius: Rekona, 2005.-123p. (in Lithuanian).
2. Eurocode 2: Design of Concrete Structures - Part 1: General Rules and Rules for Buildings. European Committee for Standartization. Pr - EN 1992 - 1, October 2001.-230p.
3. **Zaliesov, A.S.; Chistiakov, E.A.; Laricheva, I.J.** New methods of calculating reinforced concrete members at normal sections based on deformation calculation model.-Concrete and Reinforced Concrete, 5, 1997, p.31-34 (in Russian).
4. **Erich Raue.** Non-linear analysis of composite cross-sections by non-linear optimisation.-Modern Building Materials, Structures and Techniques: Abstracts of the 9th international conference held in Vilnius on May 16-18, 2007; (full paper on enclosed CD-ROM, 9p.). -Vilnius: Technika, 2007, p.434.
5. **Židonis, I.** Calculation method of stress strain of reinforced concrete members at normal sections.-Reinforced Constructions: Strength of the Concrete and Reinforced Concrete, 10.-Vilnius, 1980, p.65-72 (in Russian).
6. **Židonis, I.** A Method of calculation of parameters of stress strain state in structural components at normal sections.-Modern Building Materials, Structures and Techniques: Abstracts of the 9th Int. Conf. held in Vilnius on May 16-18, 2007; (full paper on enclosed CD-ROM, 10p.).-Vilnius: Technika, 2007, p.366-367.
7. **Židonis, I.** Mathematical description of the stress strain relation of materials of the type of concrete.-Materials Resistance (Proc. of the Conference held in 1980: Automation and Mechanization of Industrial Processes and Control).-Vilnius, 1980, p.38-39 (in Russian).
8. **Nazarenko, V.G.; Borovskih, A.V.** A deformation diagram of concrete taking into account the downward segment.-Concrete and Reinforced Concrete, 2, 1999, p.18-22 (in Russian).
9. **Židonis, I.** Stress strain state calculation of reinforced concrete beams at normal cracks.-Reinforced Constructions: Crack Resistance of Reinforced Concrete Constructions, 15. -Vilnius, 1987, p.42-49 (in Russian).
10. **Židonis, I.** Strain state parameters calculation of rational reinforced concrete beams.-Reinforced Constructions, 17. -Vilnius: Technika, 1995, p.23-33 (in Russian).
11. **Kaklauskas, G.** Integral Flexural Constitutive Model for Deformational Analysis of Concrete Structures.-Monography.-Vilnius: Technika, 2001.-140p.
12. **Kaklauskas, G.; Ghaboussi, J.** Stress strain relations for cracked tensile concrete from RC beam tests.-ASCE Journal of Structural Engineering, v.127, No1, January 2001, p.64-73.
13. **Marčiukaitis, G.** Modern problems and aeras of creation and development of building structures.- Building Structures: Development and Consolidation (Proc. of the Conf. held in Vilnius on 20 November 1998).-Vilnius: Technika, p.6-12 (in Lithuanian).
14. **Dulinskas, E.; Zabulionis, D.; Balevičius, R.** The equivalence analysis of code STR 2.05.05 and EC2 methods for in bending reinforced concrete elements.-Building Structures: Development of Building Structures in the Light of Integration into the Eurcode Zone. (Proc. of the national conference held in Vilnius on 3 March 2006).-Vilnius: Technika, 2006, p.147-155 (in Lithuanian).
15. **Erich Raue, Hans-Georg Timmler.** Non-linear analysis of deformations of reinforced concrete elements by non-linear mathematical optimisation.–Modern Building Materials, Structures and Techniques: Abstracts of the 9th Int. Conf. held in Vilnius on May 16-18, 2007; (full paper on enclosed CD-ROM, 5p.).-Vilnius: Technika, 2007, p.435.

I. Židonis

LENGVAI INTEGRUOJAMA BETONO DEFORMACIJŲ IR ĮTEMPIŲ TARPUSAVIO PRIKLAUSOMYBĖ IR JOS APRAŠYMO METODIKA

R e z i u m ė

Dažnai tenka apskaičiuoti gelžbetoninių elementų deformacijų įtempių būvį normaliniuose pjūviuose. Tai daroma įvairiose elemento apkrovimo stadijose: iki plyšių susidarymo, plyšimo, konstrukcijų su plyšiais eksploatavimo, elemento irimo stadijose.

Šių stadijų deformacijų įtempių būvio parametrams apskaičiuoti naudojamos labai sąlygiškos įtempių diagramos. Nėra kaip apskaičiuoti plyšių bei tempiamo betono aukščio virš jų. Labai netiksliai nustatomas gniuždomos betono zonos aukštis. Negalima apskaičiuoti minėtų parametrų stadijose prieš plyšių susidarymą bei prieš elementų irimą. Neturime galimybės nustatyti deformacijų įtempių būklę nekarpytų elementų plastiškumo šarnyruose. Nėra metodo nustatyti gelžbetoninių elementų deformacijų įtempių būklę pagal išmatuotus plyšių parametrus (plyšių plotį, aukštį ir atstumą tarp jų) stadijose, artimose elemento irimui.

Daug problemų galima išspręsti, naudojant realesnes, kreivines betono deformacijų įtempių diagramas. Bet jas naudoti sunku dėl integravimo problemų.

Normaliojo, sunkiojo bei smulkiagrūdžio C8/10-C90/105 stiprio klasių betono įtempiams σ_c apskaičiuoti straipsnio autorius siūlo formulę. Ją tikslinga naudoti tuomet, kai reikia integruoti kreivinę įtempių diagramą, pavyzdžiui, sudarant poveikių statinės pusiausvyros lygtis.

I. Židonis

A SIMPLE-TO-INTEGRATE FORMULA OF STRESS AS FUNCTION OF STRAIN IN CONCRETE AND ITS DESCRIPTION PROCEDURE

S u m m a r y

It is often necessary to calculate the stress strain state of reiforced concrete members at normal sections. The calculations have to be performed at different stages of loading: before cracking, at the cracking moment, in sections on the crack under service and breaking load.

For the calculations, very inaccurate diagrams of the stresses are used. There is no possibility to calculate height of the cracks and height of tensile zones of the concrete above them. The height of compressive zone of the concrete is especially inaccurate. It is impossible to calculate the mentionned parameters for the stages before cracking and before fracture. It is impossible to define these parameters in plastic hinges of the continuity beams. There is no method of stress strain state calculation of reinforced concrete members according to the measured parameters of normal cracks (height and width of the cracks and distance between the cracks) in stages close to fracture load.

It is possible to solve many of these problems using more realistic curvilinear diagrams of interrelation between strain and stress of concrete. But its use is aggravated by the integration problem.

The author of the article offers his equation for the calculation of normal stress of heavy and fine-grained concrete of strength classes C8/10-90/105. It is resonable to use this equation in cases when there is a need to integrate the curvilinear diagram of the stress when making equations of the static equilibrium of stresses.

И. Жидонис

ЛЕГКО ИНТЕГРИРУЕМАЯ ЗАВИСИМОСТЬ МЕЖДУ ДЕФОРМАЦИЯМИ И НАПРЯЖЕНИЯМИ БЕТОНА И МЕТОДИКА ЕЕ ОПИСАНИЯ

Р е з ю м е

Часто приходится рассчитывать напряженно-деформированное состояние по нормальным сечениям железобетонных элементов. Это делается в различных стадиях нагружения элемента: до образования трещин, на стадии трещинообразования, на стадиях работы элемента с трещинами, на стадии разрушения.

При расчете параметров напряженно-деформированного состояния на этих стадиях применяются сугубо условные эпюры напряжений. Нет возможности рассчитать высоту трещин и высоту растянутой зоны бетона над ними. Очень условно определяется высота сжатой зоны бетона. Нет возможности рассчитать упомянутые параметры на стадиях, бликих к образованию трещин или разрушению элементов. Нет возможности определить напряженно-деформированное состояние в пластических шарнирах неразрезных балкок. Нет метода для определения напряженно-деформированного состояния железобетонных элементов по параметрам трещин (высоте, раскрытию и расстоянию между трещинами) на стадиях, близких к разрушению.

Много проблем можно решить с применением более реальных, криволинейных диаграмм напряжений деформаций бетона. Но их применение затруднено сложностью интегрирования.

Для расчета напряжений обычного, тяжелого и мелкозернистого бетона классов прочности C8/10-C90/105 автор статьи предлагает зависимость для определения напряжении в бетоне σ_c. Эту формулу целесообразно применять в тех случаях, когда приходится интегрировать криволинейную эпюру напряжений, например, при составлении уравнений статического равновесия усилий.

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